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THE EXISTENCE PROBLEM FOR SOLUTIONS

by

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1. Introduction. In 1944 John von Neumann and Oskar Morgenstern [18] presented an extensive development of a theory of n-person cooperative games in characteristic function form. The most basic and most challenging theoretical question regarding this theory concerns whether their solution sets exist. It has been known [5] since 1967 that such solutions need not exist in the most general case of all games. However, all known counterexamples are of a rather specialized nature. And there still remains large classes of games for which this question is unanswered. So this fundamental problem continues to be among the most important and intriguing problems in cooperative game theory, as well as one that appears most difficult to solve. The answer should prove of interest in both theory and applications.

This paper briefly reviews the essential concepts of the classical von Neumann-Morgenstern (vN-M) model [18], discusses the current state of knowledge concerning the existence of solutions, lists some alternate mathematical results and unsolved problems which seem closely related to the question of existence, and finally describes a couple of the recently developed approaches to the multiperson cooperative games which may prove useful in further supporting, or even in eventually replacing, the classical vN-M theory.

2. The Model. In brief, the vN-M model consists of a function v, a set A, a preference relation dom, and a solution concept V. An n-person game (in characteristic function form) is a pair (N,v) where $N = \{1,2,\ldots,n\}$ is the set of players and v is a real-valued characteristic function on 2^N; i.e., v assigns the real number $v(S)$ to each subset S of N, and $v(\emptyset) = 0$ for the empty set $\emptyset$. The set of imputations is

$$A = \{x: \sum_{i\in N} x_i = v(N) \text{ and } x_i \geq v(\{i\}) \text{ for all } i \in N\}$$

where $x = (x_1,x_2,\ldots,x_n)$ is a vector with real components. For any nonempty $S \subset N$ let

$$x(S) = \sum_{i \in S} x_i.$$

For any x and $y \in A$ and nonempty $S \subset N$, we say that x <u>dominates</u> y <u>with respect to</u> S, denoted $x \operatorname{dom}_S y$, if and only if $x_i > y_i$ for all $i \in S$ and $x(S) \leq v(S)$. And we say that x <u>dominates</u> y, denoted $x \operatorname{dom} y$, whenever there is some S such that $x \operatorname{dom}_S y$. For any $B \subset A$ we let

$$\operatorname{Dom}_S B = \{y \in A\colon x \operatorname{dom}_S y \text{ for some } y \in B\}$$

and

$$\operatorname{Dom} B = \bigcup_S \operatorname{Dom}_S B \quad \text{for } \emptyset \neq S \subset N.$$

A subset V of A is a <u>solution</u>, in the sense of vN-M, whenever

$$V \cap \operatorname{Dom} V = \emptyset$$

and

$$V \cup \operatorname{Dom} V = A.$$

These latter two conditions are called <u>internal</u> and <u>external stability</u>, respectively; and solutions are now frequently referred to as <u>stable sets</u>.

The <u>core</u> of a game, which was first introduced explicitly by D. Gillies and L. Shapley in 1953, is defined either as

$$C = \{x \in A\colon x(S) \geq v(S) \text{ for all nonempty } S \subset N\}$$

or as

$$C = A - \operatorname{Dom} A.$$

If the characteristic function v is superadditive as assumed in the classical theory, i.e., $v(S \cup T) \geq v(S) + v(T)$ whenever $S \cap T = \emptyset$, then these two definitions coincide. However, in general they may differ and one must in such cases distinguish between these two definitions. In this paper, we will consider the latter one, in which C is the maximal elements with respect to the dominance relation; and this set clearly contains the former one.

A game (N,v) is constant-sum if and only if $v(S) + v(N-S) = v(N)$ for all $S \subset N$.

3. Nonexistence. It is known that there are games with ten or more players which have no solution V, and thus a general existence theorem for the full class of all games is impossible. In addition to the ten-person game appearing in [5] and [6], there are other unpublished examples of games without solutions. It should also be pointed out that in the published example there are only 15 vital coalitions (i.e., ones essential to any of the domination involved) in addition to the 13 coalitions (N and $\{i\}$ for $i \in N$) which are used in the definition of A. Most of the nearly one thousand other coalitions S can have their values $v(S)$ vary over some interval without effecting the nonexistence of solutions. E.g., one can vary many such values from 0 to the $v(S)$ which would make the known example into a superadditive game using the method of Gillies [3, pp. 68-69]. So there is a higher dimensional continuum in the space of all ten-person games which give rise to games without vN-M solutions. One can also include any finite number of additional players who are not involved in any essential way in the dominance, and thus obtain games with more than ten players which have no solutions.

It should be emphasized that the known counterexamples to a general existence theorem are hardly artificial creations, nor merely mathematical curiosities which are unlikely to appear in applications. As mentioned, they do involve very few vital coalitions. In addition, Shapley and Shubik [16] have shown that they can also arise in a very natural way from economic markets. Of course, one could dismiss all nonconstant-sum games as unimportant, including those known to have no solutions, since the derivation of the characteristic function from a game in normal form as suggested in [18] is only meaningful in the constant-sum case. On the other hand, the characteristic function does arise in a rather natural way for many games which are not constant-sum or are not given in normal form. Nevertheless, it seems quite possible that any game without a solution is rather degenerate in the mathematical sense that several particular coalitions must take on values related to each other. So nonexistence might well be rare in the probabilistic sense, i.e., in picking games at random from the space of all games. On the other hand, such games could easily occur in real applications.

It is very important to stress the fact that all known examples of games without solutions, as well as those with other "pathologies" as discussed in section 6 of [7], have the following properties:

(i) their cores are nonempty,

(ii) the dimension of their cores are less than or equal to $n/2$, which is much less than $n-1$, the dimension of A,

(iii) the regions $A - \mathrm{Dom}\, C$ also have dimensions less than or equal to $n/2$, and

(iv) they are not constant-sum.

So it is important to determine whether or not solutions do exist for all games which have empty cores, have full-dimensional cores, or are constant-sum. The games in (iv) form a subclass of those in (i). With respect to the constant-sum games, it should be noted that the classical vN-M theory [18] was first developed extensively for just the constant-sum case, and only later did they present the general-sum situation. So it seems essential to determine non-existence in the constant-sum case before one should completely discard this classical model on the basis of the lack of a general existence theorem. The few specialized results to date are hardly sufficient for this purpose. In addition, it is also important to resolve the existence problem for case (i), i.e., for games with empty cores. Since one could, for example, take as his solution concept the core when it is nonempty (even if it is not externally stable), and a vN-M solution in the case when the core is empty. The question of existence is still open for such a solution concept. Furthermore, in applications to economics, many classes of games (such as market games [16]) do have nonempty cores. And the core does seem like a fairly suitable solution concept in theoretical economics, as evidenced by the great number of publications on this topic. On the other hand, in many applications with empty cores, such as in "nondictatorial voting" games, other solution concepts, such as the Shapley or Banzhaf values (see [8]) have proved useful, and the latter has actually been supported by several rulings by courts in the United States of America. However, values are more in the nature of equity or fair division concepts, and there still is a need for solution concepts of a more "bargaining" or "noncooperative" type for coalitional interactions when the core is empty.

There are other undesirable properties or difficulties (see section 6 in [7]) with the classical vN-M theory in addition to the problem of existence. For example, the great multiplicity of solutions for some games, as well as the presence of only "pathological" types of solutions in some cases [15]. However, this general model still must be reckoned with until many more questions are answered, or until it is replaced by a better model in such situations. More evidence about existence of solutions is needed in the cases of games with empty cores and for those which are constant-sum. Without such knowledge one can hardly abandon this classical theory on the basis that some games are known to not have solutions.

Another indication of the rareness of known results on nonexistence of solutions, and an approach which might be useful in proving existence for games with empty cores is the following. Pick any particular game (N,v). Then let the one parameter $v(N)$ vary over all real numbers. (We are not now concerned with the fact that a superadditive v may now lose this property for smaller values of $v(N)$.) For small values of $v(N)$, A, and thus V, is the empty set. As $v(N)$ increases somewhat, A first becomes a single point and then a "small" simplex; and solutions will always exist for some such interval of $v(N)$, because the resulting games are similar to "simple" games (see [14] and [18]). As $v(N)$ continues to increase, there is generally an interval in which the core is empty and the problem of existence of solutions is unresolved. Next, there is one value of $v(N)$ at which the core is nonempty and of dimension less than $n-1$. It is known that some games do not have solutions at this <u>one particular</u> value of $v(N)$. As $v(N)$ continues to increase, the resulting games will have full-dimensional cores, and the question of existence is still open. Eventually, for all large values of $v(N)$ the core is "large" enough that it will be a solution by itself. One approach to proving existence for any game with empty core is to pick the smallest $v(N)$ at which no solution exists, and to attempt to derive a contradiction from this supposition.

It should also be stressed that all known counterexamples to existence seem to be very sensitive to changes in the values $v(S)$ of <u>many</u> of the vital coalitions in these games, especially to the value $v(N)$ of the grand coalition. Some values $v(S)$ can be altered individually or simultaneously without losing the nonexistence or other pathologies, but most often such changes drastically alter the nature or possibility of solutions in the known examples. For example, increasing only $v(N)$ in such known games creates ones with full-dimensional cores which seems to have solutions. Whereas, decreasing just $v(N)$ gives nonsuperadditive games with empty cores which do have many solutions, at least some of which are without strange pathologies. The "pyramid extensions" (see [3] and below) also frequently give rise to games with cores of full-dimension, and they seem to have a variety of solutions. For every n-person game there is an $(n+1)$-person "constant-sum extension" (see [18]), which inherits the solutions of the former, but the latter may also have other solutions as is the case with the known examples of games without solutions.

4. Related Problems. A few particular mathematical difficulties seem to repeatedly reappear in typical attempts or in the usual approaches to proving existence, as well as in efforts to construct counterexamples to disprove it. A few of these more common obstacles will be discussed in this section, since the solving of the alternate problems may prove to be major steps towards answering our primary question of existence.

First, recall that the core C of any game is contained in each of its solutions V. So a game with a nonempty core has at least some imputations which are "fixed" in any solution. In some cases one can then "trim" the core to a smaller one in such a way as to obtain a nonconvex region which is in every solution. Note that C is always convex. One then introduces additional vital coalitions which create an odd cycle of domination, i.e., one obtains $x \text{ dom } y$, $y \text{ dom } z$, and $z \text{ dom } x$, which causes the desired contradiction to the existence of any solution.

On the other hand, when $C = \emptyset$, there is domination throughout all of A, and one can normally find a great multiplicity of solutions. The inability to "pin down" elements of A which are in every solution set makes it difficult to construct games without solutions. So one very important question is:

$$\text{Is } \cap V = \emptyset \text{ for every game with } C = \emptyset?$$

The symbol $\cap V$ denotes the intersection of all solutions V for a given game. If the answer to this question is negative, then one could quite likely construct related games without any solutions. But if the reply is positive, then it may prove more difficult to find such counterexamples. One may recall that the parallel result for games with nonempty cores was the discovery of a game in which $\cap V$ was a proper superset of C, and this was the first significant step in the determination of nonexistence in the case when $C \neq \emptyset$. (See section 6 in [7].)

It does not seem essential to have $\cap V \neq \emptyset$ in order to disprove existence. Perhaps each V could be of a rather specific type; and one could construct a counterexample by means of a higher-dimensional or "moving" singularity which exists over some line or other subspace rather than at just a single point, and which acts on each of the potential solutions V. For example, the four-person (nonsuperadditive) game

$$v(1234) = 10,\ v(12) = v(34) = 6, \text{ and}$$

$$v(S) = 0 \text{ for all other } S \subset N = \{1,2,3,4\}$$

has a continuum of stable sets, but each one is a rectangle

$$V(a) = \{x \in A: x(12) = a, x(34) = 10-a\}$$

where $4 \leq a \leq 6$; and each of these is not too different from a square. Similar results hold in higher dimensions, and each such "box" is rather similar to the hypercubes used in known counterexamples. However, when one attempts to "trim" such rectangles in the case of empty cores, experience indicates that a great variety of other solutions seem to appear. For example, if one changes only the value of the coalition $\{1,3\}$ in this example to

$$v(13) = 3$$

then many solutions of various types appear for this new game in addition to the previous sets $V(a)$ trimmed down to $V(a) \cap \{x: x(13) \geq 3\}$. Such multiplicity makes it difficult to "fix on a singularity" of the type which seems necessary to construct games without solutions.

Second, many attempts to prove existence could make use of the fact that some of the sets involved are connected sets. Of course, it is well known that many individual solutions V are not connected. However, there are, for example, questions as to whether the union $\cup V$ of all solutions is connected, whether any two solutions of a pyramid game are connected to each other, or whether certain projections of each solution for particular special classes of games are connected. For example, it is not known whether or not the elementary four-person game

$$v(1234) = v(123) = v(124) = v(134) = 1, \text{ and}$$

$$v(S) = 0 \text{ for all other } S \subset N = \{1,2,3,4\}$$

has a solution which leaves a "gap" in the x_1 direction; i.e., whether there is any solution V and value a $(0 \leq a < 1)$ such that $V \cap \{x \in A: x_1 = a\} = \emptyset$. This game does have a great variety of solutions, some "pathological" and some of a simpler nature [13]. Gillies [3] introduced the idea of a <u>pyramid extension</u> of an arbitrary game by adding a player who is in every vital coalition. Dominance in such games is acyclic; and the goal is to prove existence for pyramid games, and to then show that a solution to the original game can be obtained from one for the extension. It appears that this approach, as well as the similar one of varying $v(N)$ which was mentioned in the second last paragraph of section 3, would be more successful if certain connectivity properties were present. The very definition of a solution is that it is a fixed point of the map

$$f: 2^A \to 2^A \quad \text{where} \quad f(X) = A - \text{Dom } X$$

and where 2^A stands for the class of all subsets X of A. And fixed-point theorems suggest connectivity. It should also be noted that for a game with an empty core, or lower-dimensional core, that any solution V is a set with the property that $\text{Dom } V$ is a union of open sets (open orthants actually) whose closure covers V and whose boundary contains V. This is also suggestive of topological theorems relating to connected sets. In addition, some attempts to obtain solutions as nested sequences or as fixed points in a lattice have also been made.

There has been a long-standing conjecture that the union $\cup V$ of all solution V to any game is a connected set. However, there is a recent example [9] of a twelve-person game for which this is false. On the other hand, this example has a nonempty core of dimension $n/2$, and $A - \text{Dom } C$

is also of dimension $n/2$, as has been the case for most other results in this direction. Although there are several questions related to connectivity, a major one is the following:

Is UV connected for every game with $C = \emptyset$?

A negative reply would likely lead to the construction of a game with no solution and "no" core (i.e., $C = \emptyset$). A positive answer might prove useful in proving existence for games with empty cores, or for constant-sum games. It is also of some interest to determine whether UV is connected for the case of full-dimensional core.

Third, it may well prove helpful to analyze additional special classes of games (N,v) as well as to investigate certain generalizations and variations of the classical vN-M model. In attempting to prove existence, one might gain insights by demonstrating it first for special cases. And it is usually easier to first find counterexamples in a generalization, since a theorem must then hold for a larger class of objects. It should be observed that nonexistence of solutions in the case of nonempty core was first obtained for several generalizations of the vN-M theory, as discussed in section 5 of [7], before it was determined for the classical model. There are "games without side payments" and "games in partition function form" (see [1], [17] and sections 5.3 and 5.4 in [7]) which have empty cores and no solutions. Solutions have also been studied recently for variations in the vN-M theory which alter the definition of v, A, or the dominance relation. Extensive work in this direction has, for example, been undertaken by a group in Leningrad under O. N. Bondareva, as is illustrated by the paper of D. Fink [2]. Additional investigations along these lines should prove to be beneficial to better understanding of the fundamental problems in the original vN-M theory.

There are certainly many additional problems concerning the existence and nature of solutions whose resolution would prove of interest. Knowledge of common properties or of the general nature of the set of all games without solutions would be interesting, as well as the "frequency" with which this apparently highly degenerate phenomena can occur. One would like to know the smallest value of n at which such pathologies can first appear. And one would like to construct counterexamples based on other geometrical configurations than those currently known. Additional information on the nature of solutions and their multiplicities for particular games or classes of games would be desirable.

5. Additional Models. There is clearly a real need for a new model, or at least a modified solution concept, to replace the vN-M ones for some classes of multiperson cooperative games. One can essentially abandon this original theory or its solution concept and replace them by substantially different ones; as is somewhat the case for solution concepts such as bargaining sets, kernels, nuclei, and various value theories, although the latter do begin with the characteristic function. Or one can merely modify the classical theory so that the revisions do not have the undesirable theoretical properties or are more likely to be useful in applications. Two recent approaches of the latter type which are rather close to the vN-M model and which should prove important are described briefly in this section. The first lends some support to the classical model from the applied point of view in that it illustrates that many of the same solutions arise when one takes a more "noncooperative" or dynamical approach to the characteristic function games. Whereas the second approach, which should prove useful in theory, does have existence theorems in which the proofs make use of sophisticated mathematical arguments which have been unsuccessful to date when applied to the classical situation.

One approach dates back to a suggestion of John Nash [10] where he writes that:

> "A...type of application is to the study of cooperative games... . One proceeds by constructing a model of the pre-play negotiations so that the steps of the negotiation become moves in a larger non-cooperative game describing the total situation...thus the problem of analyzing a cooperative game becomes the problem of obtaining a suitable, and convincing,...model for the negotiation."

In recent years, the noncooperative approach to the cooperative games has been pursued by Selten [12], Harsanyi [4], and Weber [19]. Some valid criticisms of the vN-M model include: its static nature, the fact that the relation $x \operatorname{dom}_S y$ seems to imply that the coalition S controls the components of x belonging to $N-S$ as well as S, and the argument that one will not oppose an imputation in a solution V with one outside of V that dominates it since it in turn is dominated by some imputation in V. The "noncooperative" approaches attempt to overcome some of these objections by developing multi-stage negotiations or bargaining schemes for such characteristic function games. For example, the model of Weber [19] begins with an arbitrary "proposed" imputation x. Then each coalition S suggests a "partial imputation" y^S defined only for the components corresponding to the players in S and with the property that "$y^S \operatorname{dom}_S x$". Some rule then picks a particular S and y^S, and then $N-S$ determines the remaining components y^{N-S} such that $y^{N-S}(N-S) = v(N) - y^S(S)$. The resulting imputation x^1 determined from y^S and y^{N-S} is a new proposal, and this process can be repeated until some stopping rule ends it or until no coalition S offers an alternate y^S to a current proposal. This bargaining process leads to certain <u>stationary</u> proposals to which no further objections are voiced. Weber [19] has determined some such <u>bargaining solutions</u> and <u>stationary sets</u> for several classes of n-person games, and interestingly they often correspond to some, but not all, of the vN-M solutions. So these interesting new approaches tend to support or reinforce many of the results obtained in the original vN-M theory.

In another approach, Roth [11] introduces some alternate solution concepts having some similarities to the vN-M solutions. He is then quite successful in obtaining existence theorems for these concepts. In section 4 we observed that a solution is a fixed point of the "undom" map $f\colon 2^A \to 2^A$ where

$$f(X) = A - \mathrm{Dom}\, X, \quad X \subset A.$$

One can compose this function f to generate the nested sequences

$$A \equiv f^0(A) \supset f^2(A) \supset \ldots \supset f^{2k}(A) \supset \ldots \supset V$$

and

$$C \equiv f(A) \subset f^3(A) \subset \ldots \subset f^{2k+1}(A) \subset \ldots \subset V$$

where $k = 0,1,2,\ldots,$ and where V can be any solution to the particular game. Such sequences have been investigated before by L. Shapley and others. Roth calls a subset L of A a <u>subsolution</u> to the game (N,v) if and only if

$$L \subset f(L)$$

and

$$L = f^2(L).$$

That is, whenever L is internally stable,

$$L \cap \mathrm{Dom}\, L = \emptyset,$$

and L is simultaneously equal to the imputations $f^2(L)$ protected by itself. The latter condition implies that

$$\mathrm{Dom}(f(L) - L) \supset f(L) - L,$$

i.e., imputations outside of L and Dom L will, as a collection, dominate each other. Any subsolution L contains the core C, and any solution V is a maximal subsolution. Roth [11] has shown that every game has at least one maximal subsolution. However, it is not yet known whether or not there is a nonempty subsolution for every game with an empty core. He has also investigated the intersection

$$L^0 = \cap L$$

of all subsolutions which he calls the essential standard, as well as the supercore

$$C^+ = \bigcup_{k=0}^{\infty} f^{2k+1}(A).$$

More analysis and experimentation is needed in order to determine the potential usefulness of these concepts in applications. On the other hand, it would prove most beneficial if analytical techniques such as Roth's could be employed more successfully in the vN-M theory as well as in other game theoretical models.

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This paper discusses the current state of knowledge on the existence of von Neumann-Morgenstern solutions (stable sets). Some alternate mathematical problems related to existence are described, and a couple of recently introduced solution concepts related to the classical model are mentioned.

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